Chapter 7: Conservation of Energy
Chapter 9: Center of Mass and Momentum
Tuesday February 17th

- Review: Potential Energy
- Review: Conservation of Energy
- Calculus method for determining work
- Example problems, *i*clicker and demos
- Chapter 9: center of mass (if time)
- Chapter 9: momentum and impulse (if time)

Mini Exam III on Thursday

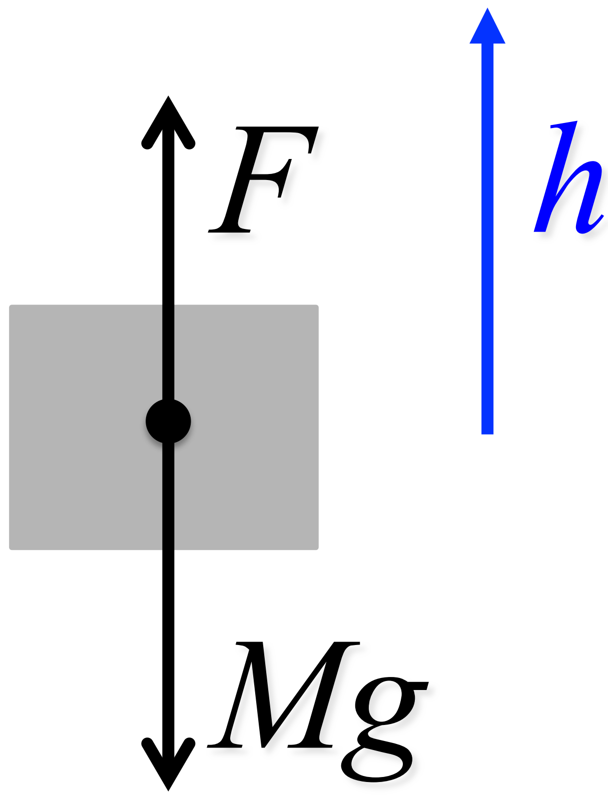
- Will cover LONCAPA #7-10 (Newton's laws and energy cons.)

Reading: up to page 113 in Ch. 7, then start of Ch. 9

Work & Potential Energy

One can define a 'Potential Energy', *U*, for ALL conservative forces as follows:

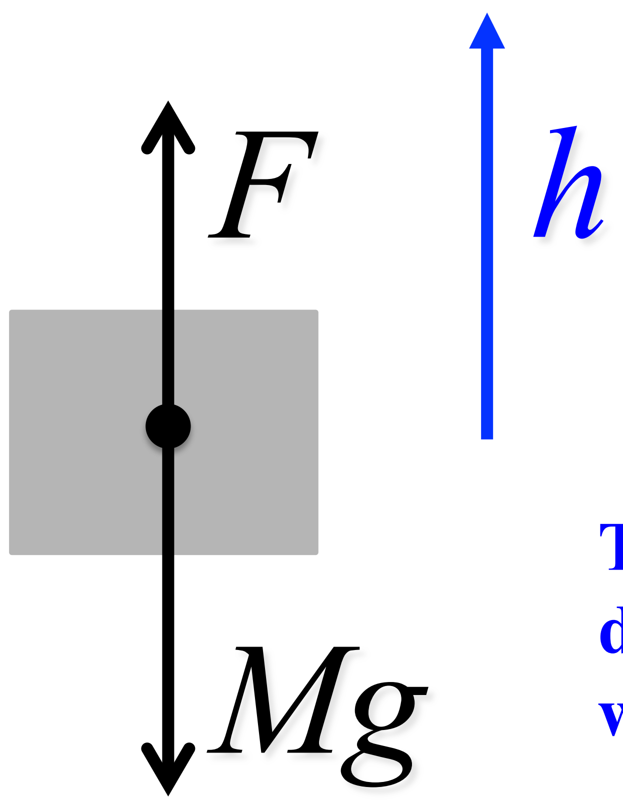
$$\Delta U = U_f - U_i$$
$$= -W_{\text{cons.}}$$
$$= -F_g \Delta x$$



Work & Potential Energy

One can define a 'Potential Energy', *U*, for ALL conservative forces as follows:

$$\Delta U = U_f - U_i$$
$$= -W_{\text{cons.}}$$
$$= -F_g \Delta x$$
$$\Delta U_g = Mgh$$



The Potential Energy change, ΔU, does not care how the height change was achieved.

Conservation of Energy

Work-Kinetic Energy theorem

$$\Delta K = K_f - K_i = W_{\mathrm{net}}$$
$$= W_{\mathrm{cons.}} + W_{\mathrm{n.c.}}$$

- We can now replace any work due to conservative forces by potential energy terms, i.e.,

$$\Delta K = -\Delta U + W_{\mathrm{n.c.}}$$

Or

$$\Delta E_{\mathrm{mech}} = \Delta K + \Delta U = W_{\mathrm{n.c.}}$$

- Here, E_{mech} is the total mechanical energy of a system, equal to the sum of the kinetic and potential energy of the system.
- If work is performed <u>on</u> the system by an external, non-conservative force, then E_{mech} **<u>increases</u>**.

Conservation of Energy

Alternatively:

$$\Delta E_{\text{mech}} = \Delta K + \Delta U = W_{\text{n.c.}}$$

$$\Rightarrow \qquad \left(K_f - K_i\right) + \left(U_f - U_i\right) = W_{\text{n.c.}}$$

Or $$K_f + U_f = \left(K_i + U_i\right) + W_{\text{n.c.}}$$

i.e. $$E_{\text{mech},f} = E_{\text{mech},i} + W_{\text{n.c.}}$$

Power

- Power is defined as the "rate at which work is done."
- If an amount of work W is done in a time interval Δt by a force, the average power due to the force during the time interval is defined as

$$P_{avg} = \frac{W}{\Delta t}$$

- Instantaneous power is defined as

$$P = \frac{dW}{dt}$$

- The SI unit for power is the Watt (W).

$$1 \text{ watt} = 1 \text{ W} = 1 \text{ J/s} = 0.738 \text{ ft} \cdot \text{lb/s}$$

$$1 \text{ horsepower} = 1 \text{ hp} = 550 \text{ ft} \cdot \text{lb/s} = 746 \text{ W}$$

$$1 \text{ kilowatt-hour} = 1 \text{ kW} \cdot \text{h} = (10^3 \text{ W})(3600 \text{ s}) = 3.60 \text{ MJ}$$

More on Power

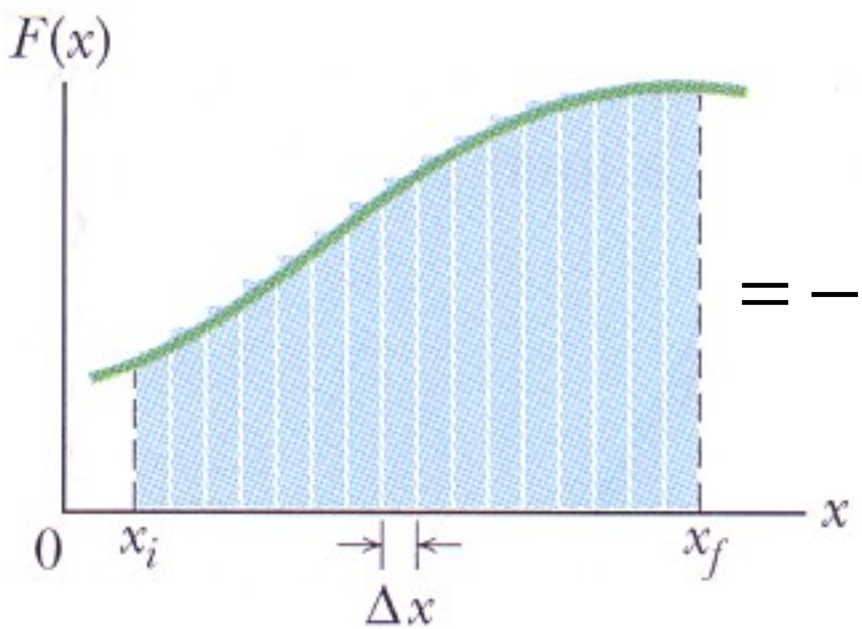
- Power is defined as the "rate at which work is done."
- If an amount of work W is done in a time interval Δt by a force, the average power due to the force during the time interval is defined as

$$P_{avg} = \frac{W}{\Delta t}$$

- Alternative definition of instantaneous power:

$$P = \frac{dW}{dt} = \vec{F}.\frac{d\vec{r}}{dt} = \vec{F}.\vec{v}$$

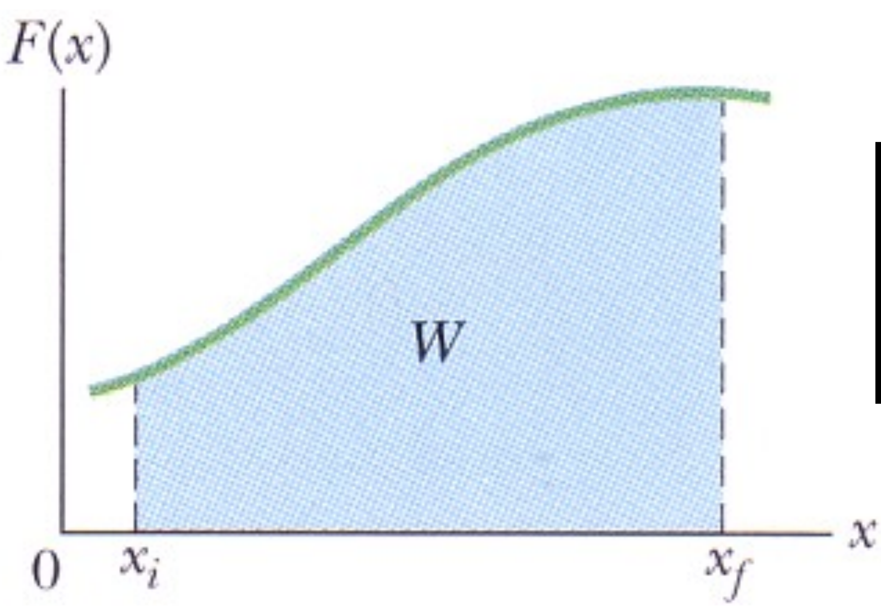
General (calculus) method for calculating Work



$$\Delta U = -W_c$$

$$= -\left(F_{c1}\Delta x + F_{c2}\Delta x + \ldots . F_{cj}\Delta x + \ldots\right)$$

$$= -\sum_j F_{cj}\Delta x$$

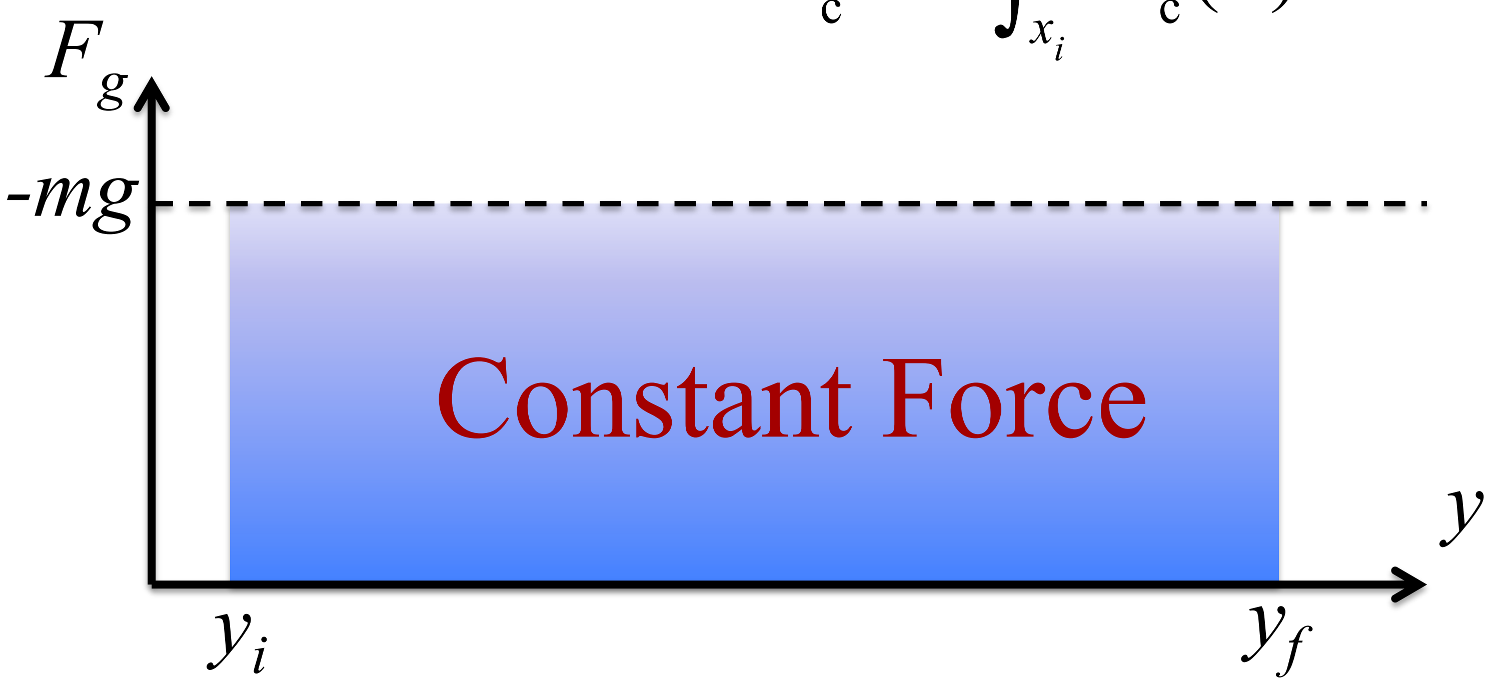


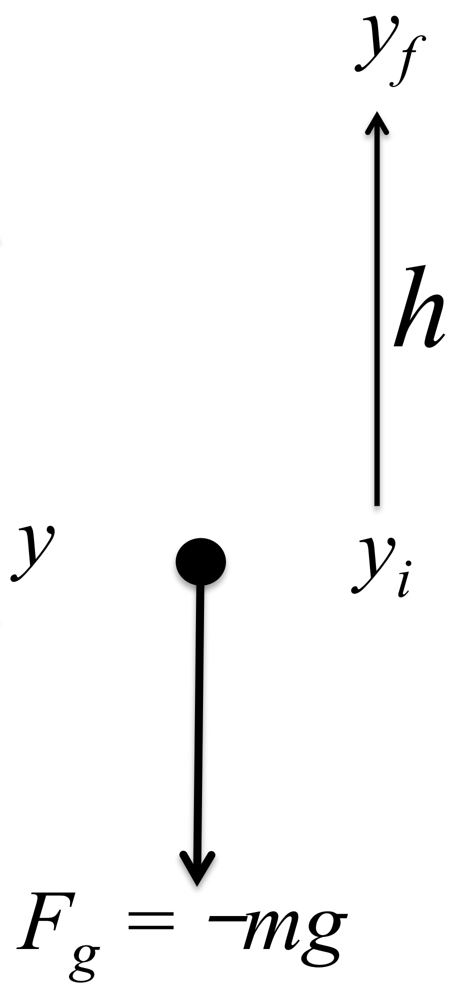
Take the limit $\Delta x \to 0$

$$\Delta U = -W_c = -\int_{x_i}^{x_f} F_c(x)\,dx$$

Gravitational Potential Energy

$$\Delta U = -W_c = -\int_{x_i}^{x_f} F_c(x)\,dx$$



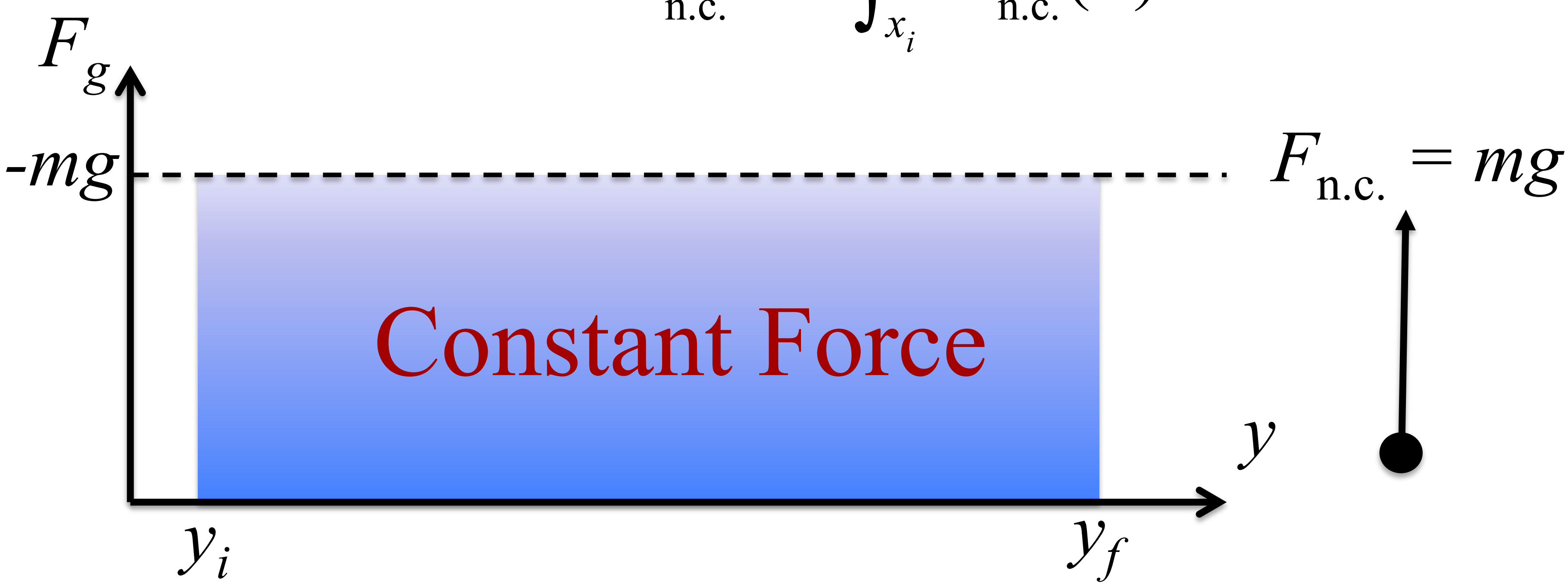


$$\Delta U = -\int_{y_i}^{y_f} (-mg)\,dy = mg\int_{y_i}^{y_f} dy$$

$$= mg\left(y_f - y_i\right) = mg\Delta y = mgh$$

Gravitational Potential Energy

$$\Delta U = +W_{\text{n.c.}} = +\int_{x_i}^{x_f} F_{\text{n.c.}}(x)\,dx$$

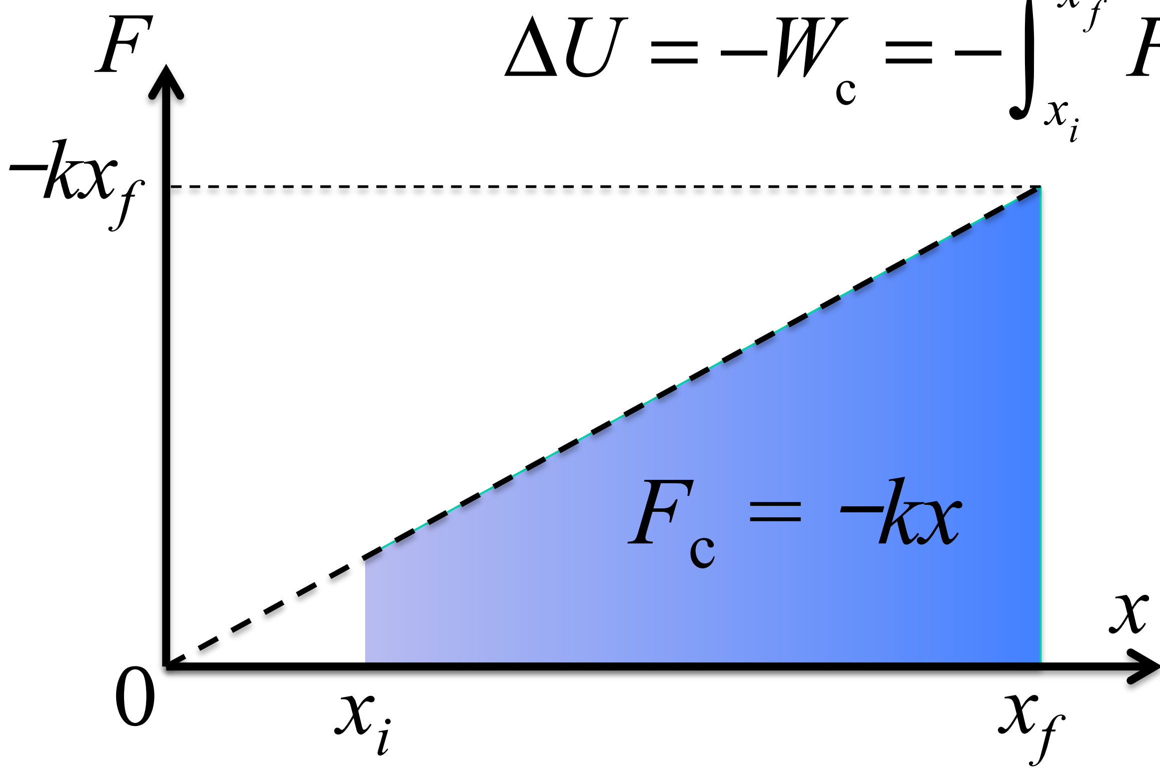


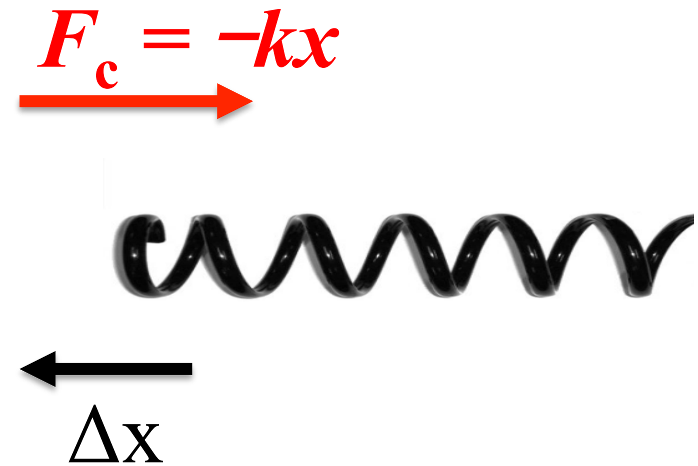
$$\Delta U = +\int_{y_i}^{y_f} (+mg)\,dy = mg\int_{y_i}^{y_f} dy$$

$$= mg\left(y_f - y_i\right) = mg\Delta y = mgh$$

Elastic/Spring Potential Energy

$$\Delta U = -W_c = -\int_{x_i}^{x_f} F_c(x)\,dx$$





$$U = -\int_0^x (-kx)\,dx = k\int_0^x x\,dx$$

$$= \tfrac{1}{2}k\left[x^2\right]_0^x = \tfrac{1}{2}kx^2$$